

Analysis to the entangled states from an extended Chaplygin gas model

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With considerations of the recently released WMAP year three and supernova legacy survey (SNLS) data set analysis that favors models similar to the Λ CDM model by possibly mild fluctuations around the vacuum energy or the cosmological constant, we extend the original Chaplygin Gas model (ECG) via modifying the Chaplygin Gas equation of state by two parameters to describe an entangled mixture state from an available matter and the rest component (which can take the cosmological constant or dark energy as in the current cosmic stage, or ‘curvature-like’ term, or radiation component in the early epoch, as various phases) coexistence. At low redshifts, the connection of the ECG model and the Born-infeld field is set up. As paradigms, we use the data coming from the recently released SNLS for the first year and also the famous 157 type Ia supernova (Ia SNe) gold dataset to constrain the model parameters. The restricted results demonstrate clearly how large the entangled degree or the ratio between the energy density parameters of the two entangled phases being. The fact that the ECG models are consistent with the observations of Ia SNe is obtained through the redshift-luminosity distance diagram, hence the ECG can be regarded possible candidates for mimicking the current speed-up expansion of our universe.

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I. INTRODUCTION

The recently released high redshift SNe (SNLS-Supernova Legacy Survey project) data set has been analyzed to show it, in better agreement with WMAP year three CMB observations, favors the Λ CDM model but with its equation of state mildly around the very -1 as for a preferred cosmological constant [1]. On the other side that the serious fine-tuning problem for the cosmological constant problem and the dark side physics of Universe that have been puzzling us across the century [2], especially the recent years discovery that our universe is undergoing an accelerating phase, maybe due to a mysterious component as coined Dark Energy, forces us to model the perplexing situation more realistically, that is we try to reconcile the simple cosmological constant by a dynamically slowing varying composite with limit case back to the economic cosmological constant. We know the observational fact that our current universe evolution is controlled by a mixture of dark energy (we know less), matter (mainly the dark matter, knowing equally poor, even not less) and radiation (knowing better except for the neutrino’s absolute mass) or curvature contribution (uncertain existence) or other possible form of fluid (not certain yet). Observations of type Ia supernova (SNe Ia) directly suggest that the expansion of the universe is accelerating with possibly powered by the dark energy, and the measurement of the cosmic microwave background (CMB) [3] as well as the galaxy power spectrum for large scale structure [4] indicate that in a spatially flat isotropic universe, about two-thirds of the critical energy density

seems to be stored in a the dark energy component with enough negative pressure responsible for the currently cosmic accelerating expansion [5]. It is clear from observations that most of the matter in the Universe is in a dark (non-baryonic) form (see, for instance, [6]). To understand the cosmic dark component physics is certainly helpful for us to better model our universe and a great challenge for temporary physicists.

The simplest candidate for the dark energy is a cosmological constant Λ , which has a specially simple pressure expression $p_\Lambda = -\rho_\Lambda$. However, the Λ -term requires that the vacuum energy density be fine tuned to have the observed very tiny value, the famous “old” cosmological constant problem. To alleviate this, many other different forms of dynamically changing dark energy models have been proposed instead of the only cosmological constant incorporated model, such as modified gravity models [7]. Usually, the equation of state (EOS) for describing dark energy can be assumedly factorized into the form of $p_{DE} = w\rho_{DE}$, where w may depend on cosmological redshift z [8], or scale factor $a(t)$ with a more complicated parametrization. The case for $w = -1$ corresponding to the cosmological constant, was thought as a border-case as named the phantom divide in [9].

Recent years a new kind of model called Chaplygin Gas model (CGM) from statistic physics with its special EOS [10] as

$$p = -\frac{A}{\rho}$$

leads to a density evolution of the form

$$\rho = \sqrt{A + \frac{B}{a^6}}.$$

It interpolates between matter at relatively early epoch and dark energy/cosmological constant at late stage.

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However, it would suffer problems when considering structure formation [11] and cosmological perturbation power spectrum.

Later, a modified Chaplygin Gas Model, by generalized Chaplygin gas model (GCGM) with an EOS as

$$p = -\frac{A}{\rho^\alpha}$$

has been discussed largely in the Ref. [12] with the motivation to overcome the original model shortcomings. It describes a broad class of universe models including CGM, for choosing different range for the uncertain parameter, α , with the energy density expressed formally as:

$$\rho = \left[A + \frac{B}{a^{3(1+\alpha)}} \right]^{1/(1+\alpha)} = [\rho_\Lambda^{\alpha+1} + \rho_m^{\alpha+1}]^{1/(1+\alpha)}, \quad (1)$$

in which both A and B are parameters and, $0 \leq \alpha \leq 1$. When one takes $\alpha = 1$, it returns to the original CGM. It is remarkable that the model interpolates like the CGM between a de Sitter universe and a dust-dominated one via an intermediate phase which is a mixture of a cosmological constant and a perfect fluid with a “soft” matter equation of state, $p = \alpha\rho$ [13]. This mixed state is not the same as conventional ones, for its non-linear mixture property in the right hand side of the Friedmann equation

$$\rho \neq \sum \rho_i = \rho_m + \rho_{DE} + \rho_k + \dots, \quad (2)$$

with these suffixes indicating possible contributions from matter, dark energy, curvature, ...

This is just one choice to extend the CGM. For the parameter A in CGM, it can be generalized, too [14]. A natural and simple consideration for A is a power-law relation with the redshift, z (in the standard cosmology $z = 1/a - 1$), e.g. $A(z) = A_0(1+z)^m$. Based on GCGM, such extension would give a model with a bigger parameter space- $\{m, \alpha\}$ (The case $m = 0$ leads back to GCGM). Directly from Eq. (1) may a potential merit of such extension that the density evolution has got a little more general form

$$\rho = \left[A_0(1+z)^m + \frac{B}{a^{3(1+\alpha)}} \right]^{1/(1+\alpha)}, \quad (3)$$

a non-linear combination between two different fluids density be palpably recognized. At the low redshift cases the first term in the square bracket can be naturally tackled as a small perturbation to the constant A_0 and the above ansatz also implies that such energy density can mimic a mixed state between a dust-dominated phase ($\rho \propto a^{-3}$) and an optional one like cosmological constant, the radiation-like ($\rho \propto a^{-4}$) or curvature-like phase ($\rho \propto a^{-2}$) determined by proper choosing the value of m . We may as well use the notion of entanglement which mostly appears in quantum mechanics to denote

the above mixed phenomenon, since energy density that depicts the stage at what the universe is has had its function like eigenstate in quantum mechanics. Especially, the modern astrophysical observations including Type Ia Supernovae, Cosmic Microwave Background, Large Scale Structure, etc. endue us with the chance to test whether this model is a plausible candidate for explaining both the earlier matter dominated phase and recent cosmic speed-up expansion of the universe. This motivates us to explore such Extended Chaplygin gas model abbreviated as ECGM during our following discussions.

The paper is arranged as follows: In Sec. II the general forms of the ECGM are introduced and in Subsec. IIA and IIB two types of the ECG models are investigated respectively; the parameter constraints and some discussions are arranged in the Sec. III. At the end, we present our conclusions with some discussions.

II. EXTENDED CHAPLYGIN GAS MODEL

A homogeneous, isotropic and flat Robertson-Walker metric can be written as

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2), \quad (4)$$

where $a(t)$ is the expansion scale factor. Under the framework of Friedman-Robertson-Walker cosmology, the global dynamic evolution of the universe is manipulated by the Friedmann equations

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3}\rho, \quad (5)$$

$$\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^2 = -4\pi G(\rho + p), \quad (6)$$

and the energy-conservation equation

$$\dot{\rho} = -3H(\rho + p),$$

or its equivalent form

$$d(\rho a^3) = -pd(a^3). \quad (7)$$

The unit convention $8\pi G/3 = c = 1$ is used in this paper, the dot denotes the derivative to the time, and the symbol H represents the Hubble parameter while z the redshift throughout. Moreover, the present scale factor a_0 is also assumed to be unit for brevity.

At the right side of the Eq. (5) there are often two conventional treatments:

- the total energy density is usually a linear addition of various fluids density, like

$$H^2 = \rho_m + \rho_{DE} + \rho_k + \dots,$$

with meanings of these suffixes as matter, Dark energy, curvature, ...

- For the mysterious dark energy, one common way treating it is to fix the matter component but proposing different models or analysis for the dark energy and other possible parts.

In this paper, we will violate these above two conventions to investigate the dark (matter and energy) fluid universe below.

From the viewpoint of the quantum field theory, the vacuum energy (or cosmological constant Λ) is the eigenvalue of the ground state of a quantum field. It is admissible to set vacuum energy a non-zero constant in the ECG model. Equivalently there are two energy fluids, the dynamical part, which can be regarded as a fluctuation and the cosmological constant part (it violates the second convention). Consequently, it deduces the following relations:

$$\rho_{ECG} = \rho_v + \rho_{dyn}, \quad p_{ECG} = p_v + p_{dyn},$$

in which the suffix, v, represents vacuum and *dyn*, dynamical contributions respectively. The EOS of the two kinds of fluids are

$$p_v = -\rho_v, \quad p_{dyn} = -\frac{A_0(1+z)^m}{\rho_{dyn}^\alpha}, \quad (8)$$

where parameters p_{dyn} and ρ_{dyn} , respectively, denote the pressure and the energy density of the dynamically evolving component of the ECG media. [For the sake of simplicity, we substitute p_{dyn} and ρ_{dyn} simply with p and ρ in the succeeding sections.]

In this model, we adopt the total effective EOS parameter $w_{T,eff}$ as defined in [15]

$$w_{T,eff}(z) \equiv -1 + \frac{1}{3} \frac{d \ln(H^2/H_0^2)}{d \ln(1+z)}. \quad (9)$$

The sound speed c_s defined by $c_s^2 = p'_{ECG}/\rho'_{ECG}$ here becomes

$$c_s^2(z) = -\alpha \frac{p}{\rho} + \frac{m}{1+z} \frac{p}{\rho'}, \quad (10)$$

for p_v and ρ_v are constants. Primes denote the derivative about the redshift z with $m \neq 0$. As $m = 0$, it returns to the GCG case. The first term in the right hand is just the sound speed for GCG model, while the second in the lower redshift can be treated as a small perturbation which we mentioned above. We will check below whether this sound speed would exceed the speed of light (see Ref. [16] etc.).

The relationship between density $\rho(a)$ and scale factor a is derived by means of Eqs. (7) and (8). That is,

$$\int d \left[a^{3(\alpha+1)} \rho^{(\alpha+1)} \right] = 3(\alpha+1)A_0 \int a^{3\alpha+2}(1+z)^{-m} da,$$

or a more applicable form

$$\int d \left[a^{3(\alpha+1)} \rho^{(\alpha+1)} \right] = 3(\alpha+1)A_0 \int a^{3\alpha-m+2} da. \quad (11)$$

In terms of the right hand side of the Eq. (11), we can mathematically divide the model into two classes according to whether the equation $m = 3(\alpha+1)$ can be satisfied or not.

A. The $m = 3(\alpha+1)$ case

In this subsection, the relation $m = 3(\alpha+1)$ holds and thus the Eq. (11) gives the following result:

$$\rho = \left[\frac{A \ln a}{a^{3(\alpha+1)}} + \frac{B}{a^{3(\alpha+1)}} \right]^{1/(\alpha+1)}, \quad (12a)$$

and its equivalent form:

$$\rho = [B - A \ln(1+z)]^{1/(\alpha+1)} (1+z)^3, \quad (12b)$$

where the B is an integration constant and A is expressed here as

$$A = 3(\alpha+1)A_0.$$

By introducing Eq. (12a) into Friedmann equation (5), we obtain

$$\int \frac{a^{1/2} da}{(A \ln a + B)^{1/2(\alpha+1)}} = \int dt,$$

which has not easily got an analytical solution. By virtue of the numerical integration, the evolution scale factor complies the expanding scenario but with a faint speed-up, which resembles that of the Einstein-de Sitter universe.

That at the cases of lower redshifts the ECG model is actually the Λ CDM model plus a correcting-term can be understood from either the Eqs. [(12a), (12b)] or the succeeding comparisons. We display for convenience the energy density of the three models of Λ CDM, GCGM, and ECGM at the Tab. I. Moreover, a trivial depict of the energy density has been put on the Fig. 1. Clearly we can see the Λ CDM can be described effectively by the ECGM with the $m = 3(\alpha+1)$ at a sufficient degree of accuracy. In particular, we can discuss the effective state parameter of this approximate model to the Λ CDM ($w = -1$) if one introduces the Eq. (12b) back to the Eq. (9). As a result, the effective state parameter is

$$w_{T,eff}(z) = -1 + \left[1 - \frac{A_0}{B - A \ln(1+z)} \right] \frac{\rho}{H_0^2}, \quad (13)$$

At low redshifts, one nontrivial point is that $A_0 = B$ leading to $w_{T,eff} = -1$. Thus the second term at the right side can be viewed as a small perturbation around -1 .

The sound speed Eq. (10) becomes

$$c_s^2(z) = -\alpha \frac{p}{\rho} \left[1 + \frac{(\alpha+1)(B - A \ln(1+z))}{\alpha A_0 - \alpha(B - A \ln(1+z))} \right],$$

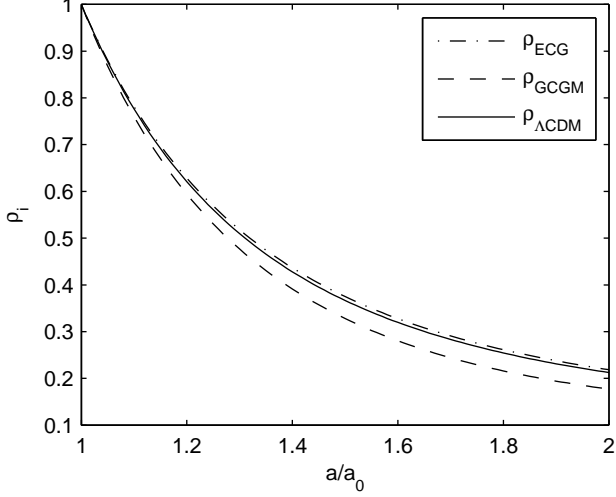


FIG. 1: The energy densities of three different models are presented at together at the low redshifts cases (We have assumed the energy density ρ_0 at present days to be unit and the vacuum density to be $\rho_\Lambda = 0.1\rho_0$).

Its property has been demonstrated on Fig. 2. Except for the two singular points, the sound speed has got its values less than 1 (i.e, light speed) satisfying the causality.

Moreover, the similarity between such class of ECG models and the Λ CDM model may indicate that the Tachyon scalar field (see reference [17]) that can depict properties of the Λ CDM is a possible source to characterize this class of ECG models as well.

B. The $m \neq 3(\alpha + 1)$ case

Now we elucidate the rest class of ECG models which persist the inequation, $m \neq 3(\alpha + 1)$. Thus, the Eq. (11) gives the following result

$$\rho = \left[\frac{A}{a^m} + \frac{B}{a^{3(\alpha+1)}} \right]^{1/(\alpha+1)}, \quad (14a)$$

or its equivalent form

$$\rho = \left[A(1+z)^m + B(1+z)^{3(\alpha+1)} \right]^{1/(\alpha+1)}, \quad (14b)$$

where A is used to denote

$$A = \frac{3(\alpha+1)A_0}{3(\alpha+1) - m}.$$

One worth noticing point lies in the reality that when $m=0$, it returns to the GCG model while to Λ CDM as $m = 3(\alpha + 1)$ or $A = 0$. The clarity that the energy density evolution relying on two nontrivial parts, the dust component (the second term in the square bracket) and the uncertain part (the first term in the square bracket) is

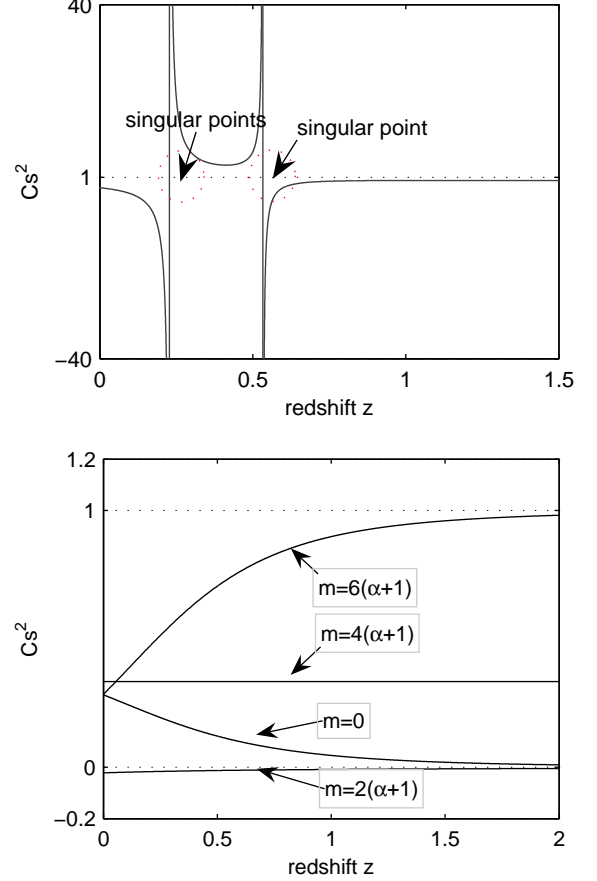


FIG. 2: The upper panel is the sound speed diagram of $m = 3(\alpha + 1)$ and the lower one is that of $m \neq 3(\alpha + 1)$. In the case of $m = 3(\alpha + 1)$, parameters take $A = 0.7$, $B = 0.3$ and $\alpha = 2/5$. In the case of $m \neq 3(\alpha + 1)$, parameters take values at Tab. II.

presented by the density formula. It also implements the density form Eq. (3) that we have expected in the Sec. I. At this case, proper choosing m values can ascertain which phase is entangled with the non-relativistic matter governed by the Eq. [(14a) or (14b)]. In the Sec. III, we will use the low redshifts data from type Ia supernova observations along with other kind of data to restrict such entangled states. However, before that we first of all formally present some connections, especially with the complex scalar field.

The EOS in this case from the Eq. (8) becomes

$$w_{eff} = -1 + \left[\frac{Am}{3(\alpha+1)} (1+z)^m + B(1+z)^{3(\alpha+1)} \right] / \rho^\alpha H_0^2. \quad (15)$$

It is easy by analogous to quantum mechanics to define the entangled degree as

$$P = \frac{2|A||B|}{A^2 + B^2}. \quad (16)$$

For clarity, $P = 0$ corresponds to no entanglement, while $P = 1$ to the maximal entanglement case.

TABLE I: This is a table which is filled with the energy density of the three different models for the purpose of comparison.

Λ CDM	GCGM	ECGM
$\rho = A(1+z)^3 + \rho_\Lambda$,	$\rho = \left[\rho_\Lambda^{(1+\alpha)} + B(1+z)^{3(1+\alpha)} \right]^{1/(1+\alpha)}$,	$\rho = [B - A \ln(1+z)]^{1/(\alpha+1)} (1+z)^3 + \rho_\Lambda$.

The sound speed from Eq. (10) turns out to be

$$c_s^2 = \frac{\alpha(1-Y)}{1 + (1+z)^{3(\alpha+1)-m}/\eta} - \frac{m(1-Y)}{3Y + 3(1+z)^{3(\alpha+1)-m}/\eta},$$

where Y takes

$$Y = \frac{m}{3(\alpha+1)}.$$

Inserting the fitted result at Tab. II, the sound speed diagram has been shown as in Fig. 2. For $m = 2(\alpha+1)$ case, the sound speed is negative which means (from the point of view for evolution of a small perturbation as a wave $\ddot{\delta} - c_s^2 \nabla^2 \delta \simeq 0$) that collapsing regions and voids get amplified. For other cases, the speeds are all positive and less than 1.

III. INVESTIGATIONS ABOUT ENTANGLEMENT

In this section, we have constrained the different ECG models characterized with corresponding values of the parameter m by using the recently released SNLS SNe and nearby dataset [18] and the famous 157 SN Ia gold dataset [19]).

A. data fitting

The dimensionless Hubble parameter for the ECG model from the Eq. (14b) reads

$$\begin{aligned} E^2(z) &= H^2/H_0^2 = \rho_{ECG}/H_0^2 \\ &= \left[\left(\frac{\Omega_X}{\Omega_M} \right)^{\alpha+1} (1+z)^m + (1+z)^{3(\alpha+1)} \right]^{\frac{1}{\alpha+1}} \Omega_M \\ &\quad + \Omega_V, \end{aligned} \quad (17)$$

where the symbols $\Omega_X = A^{1/(\alpha+1)}/H_0^2$, $\Omega_M = B^{1/(\alpha+1)}/H_0^2$, and $\Omega_V = \rho_v/H_0^2$ represent the present energy density parameters as defined by, $\Omega_i = \rho_i/H_0^2$ (i takes all possible components in question), and Ω_i reflects the relative strength of the component- i at present. These parameters together satisfy the normalization condition as favored by observations:

$$\left[\left(\frac{\Omega_X}{\Omega_M} \right)^{\alpha+1} + 1 \right]^{\frac{1}{\alpha+1}} \Omega_M + \Omega_V = 1.$$

From the analysis in the Subsec. IIB, we are aware of that as the parameter m takes values-0, $2(\alpha+1)$,

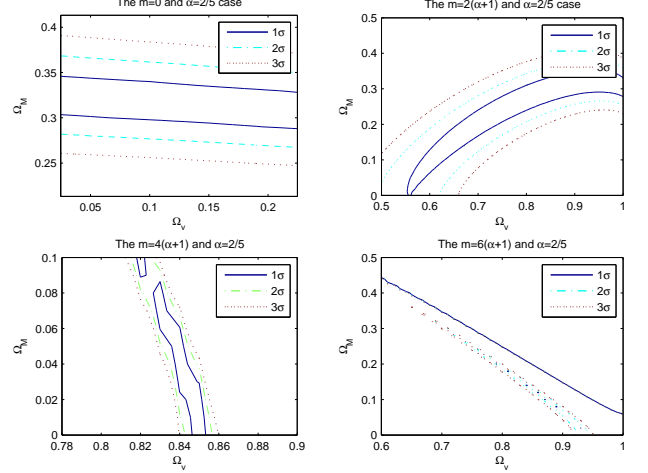


FIG. 3: The contours on the $\Omega_V - \Omega_M$ panel are in the case of $\alpha = 2/5$ for SNLS SNe and nearby data.

$4(\alpha+1)$, and $6(\alpha+1)$, the undetermined X-component of the energy density (14a) describes, respectively, cosmological constant, curvature term, radiation contribution, and stiff matter. Hereafter, symbol η is used to denote $\eta = \Omega_X/\Omega_M$, and its significance can be shown in the effective state parameter and entangled degree expression.

The most consistent values of Ω_X , Ω_M and Ω_V are usually obtained through the cosmological fitting which is actually performed in this work by minimizing

$$\chi^2 = \sum_i \frac{[\mu_{obs}(z_i) - 5 \log_{10}(d_L(z_i)/10\text{pc})]^2}{\sigma_i^2 + \sigma_{int}^2}, \quad (18)$$

where σ_i is the uncertainty in the individual distance moduli and the σ_{int} is the dispersion in supernova redshift (transformed to units of distance moduli) due to peculiar velocities. In the flat universe, the luminosity distance d_L is defined by

$$d_L(z) = \frac{1+z}{H_0} \int_0^z \frac{dz'}{E(z')},$$

We set the Hubble constant $H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$ in accordance to that appeared for SNLS SNe and nearby data in Ref. [18].

It does not lose the generality for us to take in order $\alpha = 1/2, 2/3, 4/5$ during the subsequent chi-square fit, which is compatible with the result in the Ref. [20], that is, $0.2 \lesssim \alpha \lesssim 0.6$.

As a result, the fitting values of parameters are arranged at the Tabs. [II, III, IV, V], the contours on the

TABLE II: The fitting results is in the case of $\alpha = 2/5$ for SNLS SNe and nearby data.

m	Ω_V	Ω_M	Ω_X	χ^2	η
0	0.08	0.32	0.76	150.44	2.39
$2(\alpha + 1)$	0.75	0.24	0.032	150.78	0.13
$4(\alpha + 1)$	0.85	0	0.15	150.26	∞
$6(\alpha + 1)$	0.81	0.18	0.03	150.12	0.16

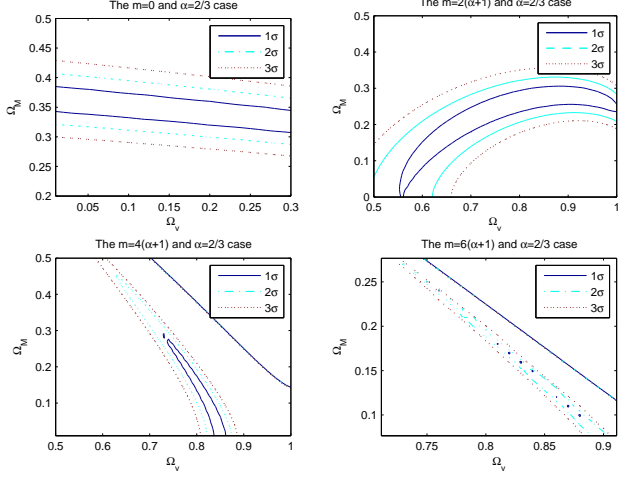


FIG. 4: The contours on the $\Omega_V - \Omega_M$ panel are in the case of $\alpha = 2/3$ for SNLS SNe and nearby data.

TABLE III: The fitting results is in the case of $\alpha = 2/3$ for SNLS SNe and nearby data.

m	Ω_V	Ω_M	Ω_X	χ^2	η
0	0.04	0.36	0.84	150.38	2.34
$2(\alpha + 1)$	0.75	0.24	0.049	150.78	0.203
$4(\alpha + 1)$	0.85	0	0.15	150.26	∞
$6(\alpha + 1)$	0.83	0.16	0.042	150.1	0.26

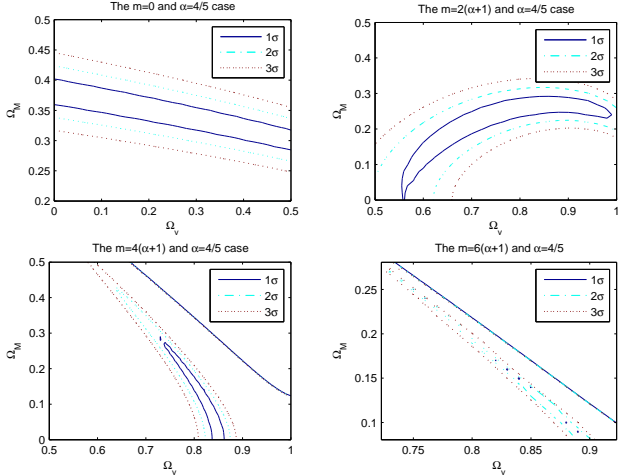


FIG. 5: The contours on the $\Omega_V - \Omega_M$ panel are in the case of $\alpha = 4/5$ for SNLS SNe and nearby data.

TABLE IV: The fitting results is in the case of $\alpha = 4/5$ for SNLS SNe and nearby data.

m	Ω_V	Ω_M	Ω_X	χ^2	η
0	0.01	0.38	0.89	150.39	2.34
$2(\alpha + 1)$	0.74	0.24	0.085	150.79	0.35
$4(\alpha + 1)$	0.85	0	0.15	150.26	∞
$6(\alpha + 1)$	0.83	0.16	0.048	150.1	0.30

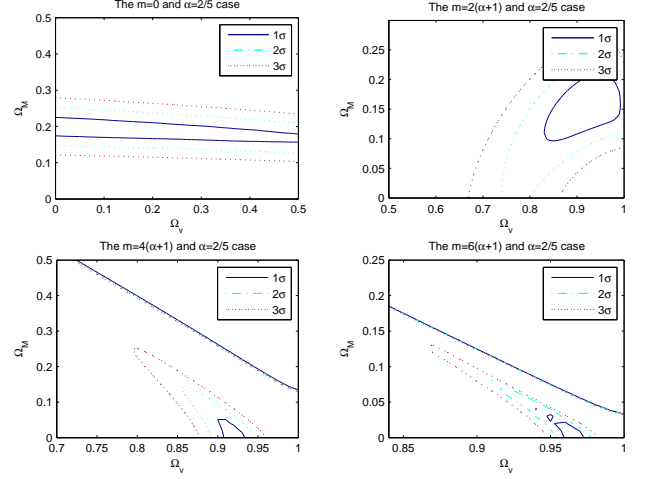


FIG. 6: The contours on the $\Omega_V - \Omega_M$ panel are in the case of $\alpha = 2/5$ for the 157 SN Ia gold data.)

$\Omega_V - \Omega_M$ panel are displayed on Figs. [3, 4, 5, 6] and at last the Hubble diagram of SNLS SNe and nearby data is showed on the Fig. 7.

B. analysis

Results from the above tables have demonstrated that the best fitted values for parameters except for η are little affected by values of α , but to the contrary, by values of m and it shows obvious distinctions between the different values of parameter m . We can rewrite the entangled degree into the form expressed by relative ratio η for

TABLE V: The fitting results from the 157 gold data is in the case of $\alpha = 2/5$.

m	Ω_V	Ω_M	Ω_X	χ^2	η
0	0	0.2	0.92	213.97	4.62
$2(\alpha + 1)$	0.87	0.13	0	216.95	0
$4(\alpha + 1)$	0.92	0	0.08	213.55	∞
$6(\alpha + 1)$	0.96	0.02	0.029	208.2	1.42

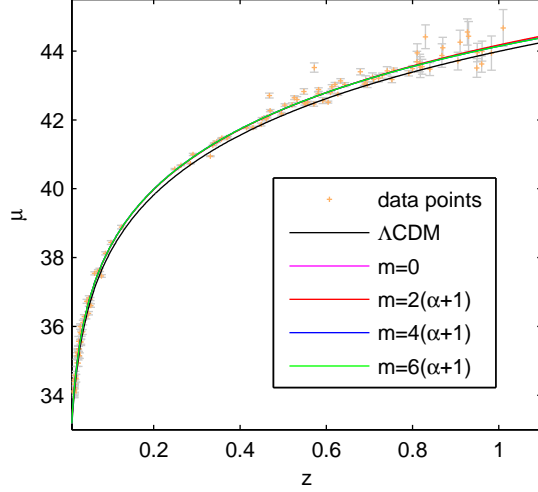


FIG. 7: Hubble diagram of SNLS and nearby SNe Ia

convenience and brevity:

$$P = \frac{2\eta^{\alpha+1}}{\eta^{2(\alpha+1)} + 1},$$

which takes zero, with meaning no entanglement, at two limit cases of either $\eta = 0$ or $\eta = \infty$ obviously.

At the case $m = 0$, the ECG model at low redshift from the Tab. II for instance has got the effective state parameter, $w_{eff} \sim -0.8$, which is consistent with the usual range $-1.3 \lesssim w_{eff} \lesssim -0.8$, meanwhile it has returned to the GCG model. In the Ref. [21] the GCG model has been considered by the use of Ia SNe with which our result of $\Omega_X \sim 0.80$ is well consistent, that is, $0.6 \lesssim \Omega_A \lesssim 0.85$. $\Omega_V \sim 0$ may have an implication that the dark side of universe favors an entangled "cosmological constant" rather than the pure vacuum energy. The entangled degree in this case is $P = 0.23$ (case $\alpha = 2/5$ for example).

At the case $m = 2(\alpha + 1)$ which corresponds to curvature term, Ω_X is a small but positive quantity. We can conclude that in the ECG universe model by data fitting it tends to be zero, that is a flat geometry and formally it is similar to the well known Λ CDM model. The effective state parameter is still $w_{eff} \sim -0.8$. The entangled degree is $P = 0$ ($\alpha = 2/5$ case for example).

At the case $m = 4(\alpha + 1)$, that $\Omega_M = 0$ corresponds to the early stage of universe at which the radiation dominated. The fitted parameters favor large value of dark energy or vacuum energy and it has even existed since the earlier universe. Its effective state parameter is $w = -0.845$. The entangled degree is $P = 0$ ($\alpha = 2/5$ for example) and there is no entanglement.

At the case $m = 6(\alpha + 1)$, the non-relativistic matter and stiff matter can coexist with a relative larger vacuum energy term. The entangled degree is $P = 0.89$ ($\alpha = 2/5$ for example).

From the Hubble diagrams as showed on the Fig. 7, the ECG models appear to be more consistent with data than the economic Λ CDM. It means that the entangled model of ECG is indeed an excellent alternative to explain the currently cosmic expansion speed-up. It is interesting to take the case of $m = 0$ into consideration, which depicts a cosmological constant entangled with the matter phase similar to Λ CDM model, but may help to overcome, as suggested in the Ref. [22] the coincidence problem, which is the fatal flaw in the Λ CDM model.

In this work with considerations of recent observational SNLS and WMAP year three dataset analysis we purposely investigate a more practical model, an extended Chaplygin gas model or we may say a Λ plus entangled CDM model in which the entangled X-component can be a radiation term, or curvature contribution, or stiff matter or constant term in special cases. Actually in the real cosmos those contributions may all exist but make different effects by taking accordingly different fractions in universe evolution stages.

IV. CONCLUSION AND DISCUSSION

In this paper, in order to further discuss the properties of dark energy which is used to explain currently cosmic accelerating expansion, we have extended the *Chaplygin gas* model by replacing the parameter, A , with a possibly variable term. Governed by the Friedmann equations, it shows a possibility to describe an entangled state or unification between a matter phase and any other phases like an entangled cosmological constant, curvature, stiff matter term or the radiation contributions, rather than only the vacuum energy component, which is the character of the GCG model.

When the relation $m = 3(\alpha + 1)$ holds, one finds that the ECG model can be treated as the Λ CDM at the low redshift cases. On another side, as the inequation, $m \neq 3(\alpha + 1)$, exists, the ECG model realizes what we have expected about the extension to GCG model. Further, the likelihood function is used to check whether such modification is reasonable. With the fitted results and analysis on the above sections, we can not exclude the possibility that there are indeed some other phases which can successfully entangle with the matter phase to determine the fate of our universe evolution altogether.

The fact that the GCGM which is deduced for the phenomenological reasons can be interpreted from a brane-world view or the Born-Infeld theory as shown below. The field ϕ and the energy density are related by the expression

$$\phi^2(\rho) = \rho^\alpha (\rho^{1+\alpha} - A)^{(1-\alpha)/(1+\alpha)}, \quad (19)$$

with the Lagrangian density as

$$\mathcal{L}_{GBI} = -A^{1/(1+\alpha)} \left[1 - (g^{\mu\nu} \theta_{,\mu} \theta_{,\nu})^{(1+\alpha)/2\alpha} \right]^{\alpha/(1+\alpha)}, \quad (20)$$

(more details can be found in the Ref. [23]). Likewise, it is possible to construct such relation corresponding to the ECG model. The Lagrangian density for a massive complex scalar field, Φ , is

$$\mathcal{L} = g^{\mu\nu} \Phi_{,\mu}^* \Phi_{,\nu} - V(|\Phi|^2),$$

which is suggestive in the Ref. [24], and the field can be expressed in terms of its mass, m , as $\Phi \equiv (\phi/\sqrt{2}m) \exp(-im\theta)$. Adopting the method in the Ref. [23], we here display the Lagrangian density for the ECG model.

The density and pressure have had the relations as

$$\rho = \frac{\phi^2}{2} V' + V, \quad p = \frac{\phi^2}{2} V' - V, \quad (21)$$

where $V = V(\phi/2)$ and $V'(x) = dV/dx$. The resulting Lagrangian density for GCGM which may have got a brane connection is

$$\mathcal{L}_{GBI} = -A^{1/(1+\alpha)} \left[1 - (g^{\mu\nu} \theta_{,\mu} \theta_{,\nu})^{(1+\alpha)/2\alpha} \right]^{\alpha/(1+\alpha)}. \quad (22)$$

At the cases of low redshifts, it is admissible that we just replace the constant parameter, A , in the Eq. (22) by the possibly variable expression, $A_0(1+z)^m$, to obtain the Lagrangian density for the ECG model. In another way, from the Eqs. [(8) and (21)] we can derive

$$(1+z)^m = -\frac{1}{A_0} \left(\frac{\phi^2}{2} V' - V \right) \left(\frac{\phi^2}{2} V' + V \right)^\alpha. \quad (23)$$

Evidently, it suffices to write the Eq. (22) into the form

$$\mathcal{L}_{GBI} = -V_{ECG} \left[1 - (g^{\mu\nu} \theta_{,\mu} \theta_{,\nu})^{(1+\alpha)/2\alpha} \right]^{\alpha/(1+\alpha)}, \quad (24)$$

where V_{ECG} is an effective potential function. When the $\alpha = 1$ it still can reproduce the Born-Infeld Lagrangian density

Observational cosmology has challenged our naive physics models, and with the anticipated advent of more precious data we will have the chance to understand or uncover the universe mysteries by more practical modelling. Quite possibly we will get more hints to unveil the cloudy cosmological constant puzzle and test whether there are really the mixed states or unified dark energy and matter or dark fluid in reasonable universe evolution.

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